Q1 Linear System

Wednesday, July 18, 2012 4:57 PM

One requirement for a linear system is that

"proportional changes in the input give the same proportional changes in the output."

In particular, if $x = 1$ corresponds to $y = 12$,

then $x = 1 \times 2$ should correspond to $y = 12 \times 2 = 24$.

(Doubling the input cause the output to double.)

In our case, we have $y = 2x + 10$.

So, if $x = 1$, $y = 2 \times 1 + 10 = 12$.

For linear system, when $x = 2$, we expect y to be 24.

However, by its definition, when $x = 2$, our system gives

$$y = 2 \times 2 + 10 = 14 \neq 24.$$

Therefore, the system is not linear.

Q2 Time Delay

Wednesday, July 18, 2012 5:17 PM

$$x(t) = 10\cos(2\pi f_c t)$$

$$y(t) = 10\cos(2\pi f_c t - \theta) = 10\cos\left(2\pi f_c \left(t - \frac{\theta}{2\pi f_c}\right)\right)$$

In the lecture, we use τ to denote propagation delay. → $\boxed{\text{delay}}$

The delay is caused by the propagation time of the signal.

Recall that the amount of time delay can be calculated from

$$\text{delay} = \frac{\text{distance}}{c}$$

c ← speed of light.

Therefore, "one possible" distance value is

$$\text{distance} = c \times \text{delay} = c \times \frac{\theta}{2\pi f_c} = \lambda_c \frac{\theta}{2\pi}$$

λ_c ← wavelength of the carrier

$$= \cancel{3}\times 10^8 \times \frac{\cancel{\pi}/\cancel{4}\ 2}{2\cancel{\pi}\times 7\times 10^6} = \frac{100}{28} \approx 3.57\ \text{m}$$

The calculation above gives only one possible distance value
Because the cosine is periodic, there can be other solutions.
In particular,

$$\cos(2\pi f_c t - \theta) = \cos(2\pi f_c t - \theta + 2\pi k) \quad \text{for any integer } k.$$

So, what we should do is to consider

$$\cos(2\pi f_c t - \theta + 2\pi k) = \cos\left(2\pi f_c \left(t - \frac{\theta}{2\pi f_c} + \frac{k}{f_c}\right)\right)$$

In which case, the amount of time delay could be

$$\frac{\theta}{2\pi f_c} - \frac{k}{f_c} \quad \text{for any integer } k.$$

The corresponding possible values of distance are

$$d = \frac{c}{f_c}\underbrace{\left(\frac{\theta}{2\pi} - k\right)} = \lambda_c\left(\frac{\theta}{2\pi} - k\right)$$

This "c" is a subscript to emphasize that this is the wavelength of the carrier.

wavelength of the carrier.

The distance is a positive quantity.

So, we need $k < \frac{\theta}{2\pi} = \frac{\pi/6}{2\pi} = \frac{1}{12}$.

In other words, k can be $0, -1, -2, -3, \ldots$

(i)
The value of k which corresponds to the minimum value of distance is $k=0$. The minimum distance is

$$d = \frac{c}{f_c}\frac{\theta}{2\pi} = 3.57 \text{ m}.$$

(ii) Other possible values of the distance are

$$d = \frac{c}{f_c}\left(\frac{\theta}{2\pi} - k\right) \quad \text{for } k = -1, -2, -3, \ldots$$

$$= \frac{c}{f_c}\left(\frac{\theta}{2\pi} + n\right) \quad \text{where } n = 1, 2, 3, \ldots$$

$$= 3.57 + 42.86n \quad \text{where } n = 1, 2, 3, \ldots$$

Q3 DSB-SC

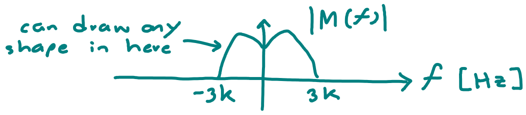
Tuesday, September 15, 2015 12:28 PM

$$v(t) = y(t) \times \cos(2\pi f_c t) \overset{y(t)=x(t)}{=} x(t) \times \cos(2\pi f_c t) \overset{x(t) = m(t)\times 3\cos(2\pi f_c t)}{=} 3\, m(t) \cos^2(2\pi f_c t)$$

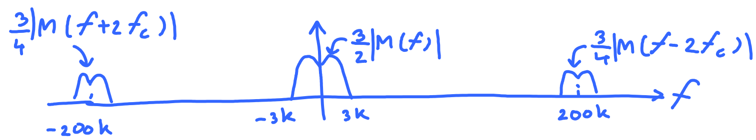
$$= 3\, m(t) \times \frac{1}{2}\left(1 + \cos(2\pi(2f_c)t)\right) = \frac{3}{2} m(t) + \frac{3}{2} m(t) \cos(2\pi(2f_c)t)$$

$V(f) = \frac{3}{2} M(f) + \frac{3}{4} M(f - 2f_c) + \frac{3}{4} M(f + 2f_c)$. Here, $f_c = 100$ kHz

$M(f)$ is assumed to be band-limited to $B = 3$ kHz.
Therefore, $M(f) = 0$ for $|f| > 3$ kHz :



Given the above picture of $|M(f)|$, we now draw the corresponding $|V(f)|$



(a) To eliminate the terms $\frac{3}{4} M(f - 2f_c)$ and $\frac{3}{4} M(f + 2f_c)$, we set

$$H_{LP}(f) = 0 \quad \text{for} \quad |f| > 200k - 3k = 197 \text{ kHz}.$$

To allow $\frac{3}{2} M(f)$ to pass through we set

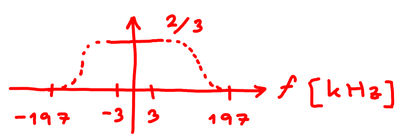
$$H_{LP}(f) = c \quad \text{for} \quad |f| < 3\text{kHz} \quad \text{for some constant } c.$$

With such $H_{LP}(f)$, we get $\hat{m}(t) = c \times \frac{3}{2} m(t)$.

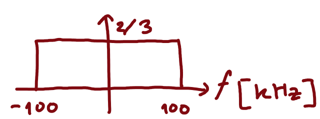
Because we need $\hat{m}(t) = m(t)$, we have to set $\frac{3}{2} c = 1 \Rightarrow c = \frac{2}{3}$.

Combining the observations above, we conclude that

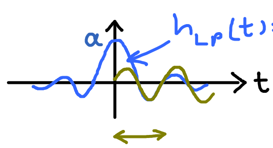
$$H_{LP}(f) = \begin{cases} \frac{2}{3}, & |f| \leq 3\text{kHz}, \\ 0, & |f| \geq 197 \text{ kHz}, \\ \text{"any"}, & \text{otherwise}. \end{cases}$$



An example would be $H_{LP}(f) = \begin{cases} 2/3, & |f| \leq 100 \text{ kHz}, \\ 0, & \text{otherwise} \end{cases}$



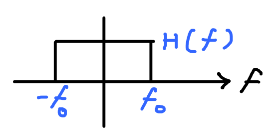
(b)



$h_{LP}(t) = \alpha \operatorname{sinc}(\underbrace{\beta t}_{= 2\pi\left(\frac{\beta}{2\pi}\right)t})$

$\frac{\beta}{2\pi}$ → freq. of the sine wave

So, period $= \frac{2\pi}{\beta}$



period $= \frac{2\pi}{\beta}$

$\frac{1}{\text{width}} = \frac{\text{period}}{2} = \frac{\pi}{\beta}$

(of the rectangular pulse)

width $= \frac{\beta}{\pi}$

width $= 2f_0$

$f_0 = \frac{\beta}{2\pi}$

Observation:

the frequency of

Area $= h_{LP}(0) = \alpha$

height $\times\, 2f_0 = \alpha$

height $= \frac{\alpha}{2f_0} = \frac{\alpha}{2 \times \frac{\beta}{2\pi}} = \pi \frac{\alpha}{\beta}$

rectangular pulse; 2π height $\times 2f_0 = \alpha$

width $= \frac{\beta}{\pi}$

Observation: the frequency of the sine wave in the sinc function gives the boundaries of the rectangular function in another domain

$$\text{height} = \frac{\alpha}{2f_0} = \frac{\alpha}{2\times\frac{\beta}{2\pi}} = \pi\frac{\alpha}{\beta}$$

the gain of the LPF

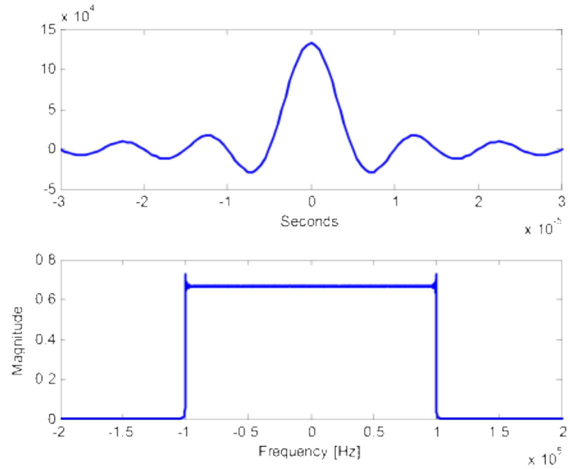
From the previous part, we need $3 \leq f_0 < 197$ kHz.

$$\text{gain} = \frac{2}{3} \Rightarrow \pi\frac{\alpha}{\beta} = \frac{2}{3} \Rightarrow \alpha = \frac{2\beta}{3\pi}.$$

So, first we choose f_0. Then, we have $\beta = 2\pi f_0$ and $\alpha = \frac{2\beta}{3\pi}$.

An example would be $f_0 = 100$ kHz $\Rightarrow \beta = 2\times10^5\pi$ rad/s and $\alpha = \frac{2}{3\cancel{\pi}}\times 2\times10^5\cancel{\pi} = \frac{4}{3}\times10^5$

Use MATLAB to check the answer above:

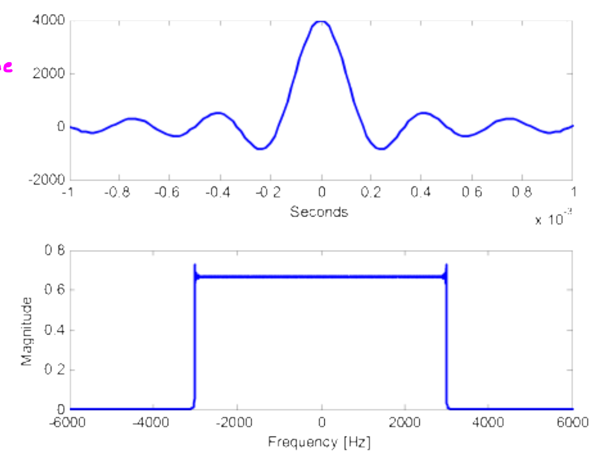


plotspect gives the plot in freq. domain that is the same as our answer in part (a).

Alternatively, one may use $f_0 = B = 3$ kHz. Then, $\beta = 2\pi f_0 = 2\pi\times3k = 6\times10^3\pi$ rad/s

$$\alpha = \frac{2}{3\pi}\beta = \frac{2}{\cancel{3}\cancel{\pi}}\times\overset{2}{\cancel{6}}\times10^3\cancel{\pi} = 4000.$$

Use MATLAB to check the answer above:



Q4 Square MODEM

Thursday, July 14, 2011 2:22 PM

(a) The process here simply repeats the analysis of the square modulator discussed in class.

$x(t) = A_c m(t) \xrightarrow{\mathcal{F}} X(f) = A_c M(f)$. So, $X(f)$ is also bandlimited to B.

$u(t) = x(t) + \sqrt{2}\cos(\omega_c t)$ $\quad \omega_c = 2\pi f_c$

$v(t) = u^2(t) = \left(x(t) + \sqrt{2}\cos(\omega_c t)\right)^2 = x^2(t) + 2\sqrt{2}\,x(t)\cos(\omega_c t) + \underbrace{2\cos^2(\omega_c t)}_{1+\cos(2\omega_c t)}$

$= \underbrace{\left(1 + x^2(t)\right)}_{\xrightarrow{BPF}\ 0} + 2\sqrt{2}\,x(t)\cos\omega_c t + \underbrace{\cos(2\omega_c t)}_{\xrightarrow{BPF}\ 0}$

Note 1: $x^2(t) \xrightarrow{\mathcal{F}} X(f) * X(f)$. So, $x^2(t)$ is bandlimited to $2B$.
Because $f_c \gg B$, the spectrum of $x^2(t)$ will not be in the passband of the BPF which centers around f_c.

Note 2: The term $\cos(2\omega_c t)$ is at frequency $2 \times f_c$ which again is outside the passband of the BPF.

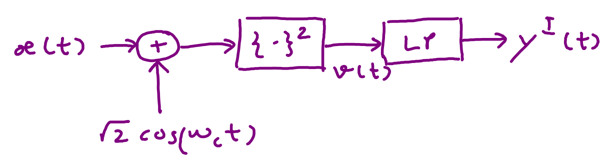
$y(t) = BPF\{v(t)\} = 2\sqrt{2}\,x(t)\cos\omega_c t = 2\sqrt{2}\,A_c\, m(t)\cos\omega_c t$

Alternatively, in class, we found that if the gain of the filter is "g" and the amplitude of the carrier is c, then $y(t) = 2c \times g \times (\text{input signal}) \times \cos(\omega_c t)$

Here, $c = \sqrt{2}$, $g = 1$, and input signal $= A_c m(t)$.

Therefore, $y(t) = 2\sqrt{2}\,A_c\, m(t)\cos(\omega_c t)$.

(b) Let $x(t) = A_c m(t)\sqrt{2}\cos(\omega_c t)$



$v(t) = \left(x(t) + \sqrt{2}\cos(\omega_c t)\right)^2 = 2\cos^2(\omega_c t)\left(A_c m(t) + 1\right)^2$

$= 1 + \cos(2\omega_c t)\,\underbrace{\left(\underbrace{A_c^2 m^2(t)}_{\text{band-limited to } 2B} + 1 + 2A_c\underbrace{m(t)}_{\text{band-limited to } B}\right)}_{\text{Define this part as } g(t)} = g(t) + \underbrace{\cancel{g(t)\cos(2\omega_c t)}}_{\xrightarrow{LPF}\ 0}$

Note 1: $g(t)$ is bandlimited to $2B$
because all of its terms are band limited to
So only some parts of it will pass through the LPF.

Note 2: $g(t)\cos(2\omega_c t)$ is centered @ $2f_c$ and therefore will not pass thought the LPF.

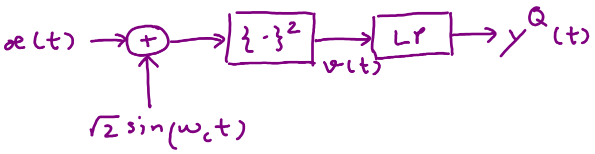
$$y^I(t) = LPF\{v(t)\} = LPF\{g(t)\} = 1 + 2A_c m(t) + LPF\{A_c^2 m^2(t)\}$$

This term has spectrum beyond IB so, only a portion of it will pass through the LPF.

$y^I(t)$ is **not** proportional to $m(t)$.

Hence, this block diagram **does not work** as a demodulator.

(c) Let $x(t) = A_c m(t)\sqrt{2}\cos(\omega_c t)$ as in part (b).



New block diagram

$$v(t) = \left(x(t) + \sqrt{2}\sin(\omega_c t)\right)^2 = 2\left(A_c m(t)\cos(\omega_c t) + \sin(\omega_c t)\right)^2$$

$$= 2\left(A_c^2 m^2(t)\cos^2(\omega_c t) + A_c m(t)\underbrace{\cos(\omega_c t)\sin(\omega_c t)} + \sin^2(\omega_c t)\right)$$

$2\cos\beta\sin\beta = \sin(2\beta)$

$$= 2\left(A_c^2 m^2(t)\cos^2(\omega_c t) + \underbrace{\sin^2(\omega_c t)}_{=1-\cos^2\omega_c t}\right) + A_c m(t)\sin(2\omega_c t)$$

$$= 2\left(\left(A_c^2 m^2(t) - 1\right)\cos^2(\omega_c t) + 1\right) + A_c m(t)\sin(2\omega_c t)$$

$$= 2 + \left(A_c^2 m^2(t) - 1\right)\left(1 + \underbrace{\cos(2\omega_c t)}_{\xrightarrow{LPF} 0}\right) + A_c m(t)\underbrace{\sin(2\omega_c t)}_{\xrightarrow{LPF} 0}$$

$$y^Q(t) = 2 + LPF\{A_c^2 m^2(t)\} - 1 = LPF\{A_c^2 m^2(t)\} + 1$$

The output alone is far from being proportional to $m(t)$. So, this block diagram also does not work as a demodulator.

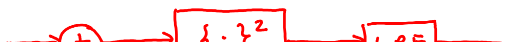
(d) Observe that

$$y^I(t) - y^Q(t) = 2A_c m(t)$$

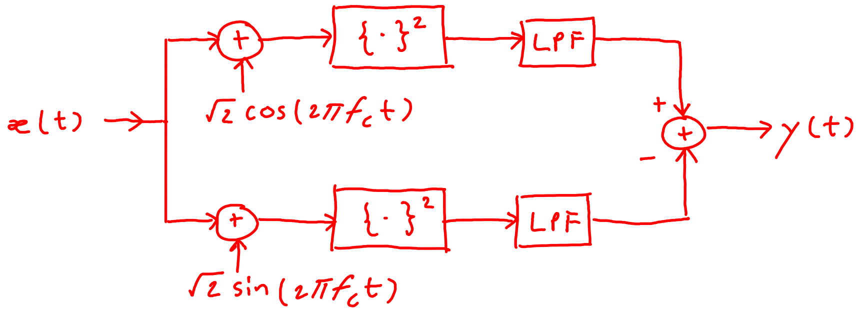
which is the desired output of a successful DSB-SC demodulator.

$y^I(t)$ from (b), $y^Q(t)$ from (c)

Hence, the following block diagram would work:



Hence, the following block diagram would work:



Q5 Cube Modulator

Thursday, July 14, 2011 2:11 PM

(a) $y(t) = \left(m(t) + \sqrt{2}\cos(2\pi f_0 t)\right)^3 = m^3(t) + 3m^2(t)\sqrt{2}\cos\omega_0 t + \underbrace{3m(t)\, 2\cos^2\omega_0 t}_{} + \underbrace{(\sqrt{2})^3\cos^3(\omega_0 t)}_{}$

$= 3m(t)\left(1+\cos 2\omega_0 t\right)$

$= 3m(t) + 3m(t)\cos(2\omega_0 t)$

$= \frac{3}{\sqrt{2}}\cos(\omega_0 t) + \frac{1}{\sqrt{2}}\cos(3\omega_0 t)$

$$\left\{\begin{aligned} 2\cos^2(\theta) &= 1+\cos(2\theta) \\ 2\cos^3(\theta) &= \cos\theta + \cos\theta\cos 2\theta \\ &= \cos\theta + \tfrac{1}{2}\cos\theta + \tfrac{1}{2}\cos 3\theta \\ &= \tfrac{3}{2}\cos\theta + \tfrac{1}{2}\cos 3\theta \end{aligned}\right\}$$

We want $z(t) = m(t)\sqrt{2}\cos(\omega_c t)$. We see that the only term in $y(t)$ that has the form "constant $\times$ m $\times$ $\cos(\ )$" is $3m(t)\cos(2\omega_0 t)$.

Therefore, we will center the passband to cover this part and adjust the gain to make the output the same as $z(t)$.

In particular, we need to make $2f_0 = f_c$. So, $f_0 = f_c/2$.

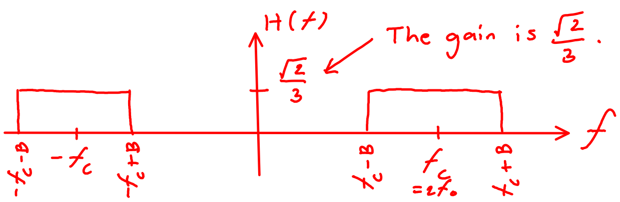
Let $H_{BP}(f) = \begin{cases} g, & |f-f_c| \le B, \\ g, & |f+f_c| \le B, \\ 0, & \text{otherwise.} \end{cases}$

Then, $z(t) = \underbrace{g \times 3}\, m(t)\cos(2\omega_0 t)$

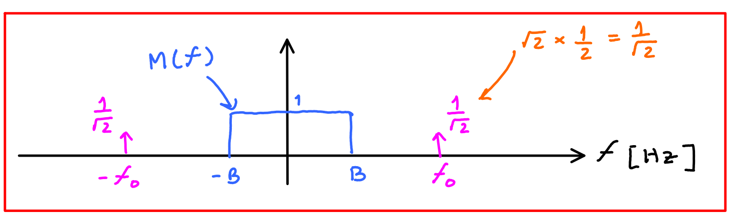
$\downarrow$

we need $g \times 3 = \sqrt{2} \Rightarrow g = \frac{\sqrt{2}}{3}$

The plot of $H(f)$ is given below:



(b.i) $x(t) = m(t) + \sqrt{2}\cos(2\pi f_0 t)$



(b.ii) From (a), we have

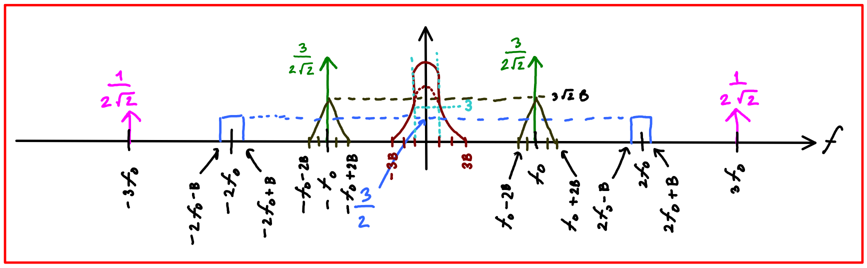
$y(t) = m^3(t) + 3m(t) + 3\sqrt{2}\, m^2(t)\cos(\omega_0 t) + 3m(t)\cos(2\omega_0 t) + \frac{1}{\sqrt{2}}\cos(3\omega_0 t)$

$+ \frac{3}{\sqrt{2}}\cos(\omega_0 t)$

Without trying to make and accurate plot for $m^3(t)$, we know that it is band limited to $3B$.

If you want to know the shape of $M(f) * M(f) * M(f)$, you can try plotting it in MATLAB using this code:

```
w = ones(1,10);
w2 = conv(w,w);
w3 = conv(w2,w);
plot(w3)
```



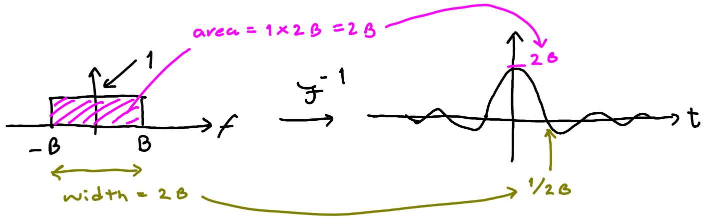
(c) $z(t) = m(t)\sqrt{2}\cos(2\pi f_c t)$. Therefore, to plot $z(t)$, first we need to find $m(t)$.

Because $m(t)$ is not specifed, we can try any $m(t)$ that is band-limited to B.

Of course, the difficult part is to find $m(t)$ that satisfies the condition.

An obvious choice would be the one that is defined in part (b).

If we find $m(t)$ from the $M(f)$ provided in part (b).

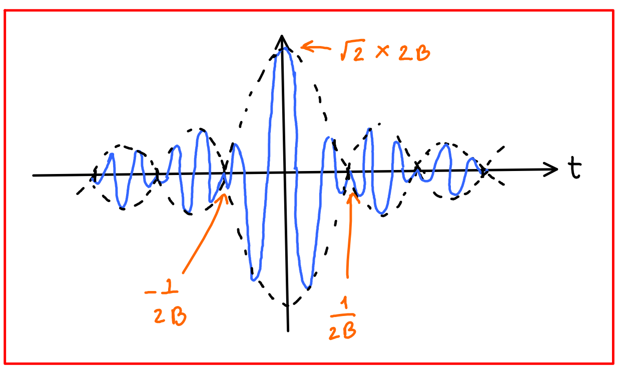


$z(t)$ is the above sinc. function multiplied by $\sqrt{2}\cos(2\pi f_c t)$.

Because $f_c \gg B$, we know that $\frac{1}{B} \gg \frac{1}{f_c}$

period of sine inside sinc   period of cosine

So, the |sinc| function becomes the envelope of the cosine carrier.



Of course, we can try to work with a different $m(t)$.

For example, $m(t) = \cos(2\pi f_0 t)$ where $f_0 \leq B$ would also satisfies the condition of being band-limited to B.

Again, because $f_c \gg B$, we also know that $f_c \gg f_0$